A Markov Regime Switching GARCH Model with Realized Measures of Volatility for Optimal Futures Hedging

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Abstract

Futures contracts are important instruments for managing the price risk exposure of spot portfolios. Over years, a number of studies have employed multivariate generalized autoregressive conditional heteroscedasticity (GARCH) models for managing the price risk, whereas the recent literature further indicates that utilizing either the Markov regime switching (MRS) or the realized volatility (RV) techniques on traditional GARCH hedging can help improving the hedging performance. This study contributes to this line of research by developing, for the first time, a multivariate MRS-GARCH model with realized measures of volatility (MRS-GARCH-X) for hedge ratio estimation, which itself is more flexible and/or informative in capturing the joint distribution of spot and futures than the existing models with stand-alone technique. To justify the performance of MRS-GARCH-X hedging, the NASDAQ 100 data are obtained for the investigations. Empirical results indicate that the MRS-GARCH-X hedging exhibits good in-sample and out-of-sample performance in terms of both criteria of variance reduction and utility growth, illustrating the statistical and economic benefits of combining the techniques of time-variation, state-dependency, and precise RV for effective futures hedging.

Keywords: multivariate GARCH model; Markov regime switching; realized volatility; futures hedging; hedging effectiveness.

JEL classification: C32, C58, G11.

INTRODUCTION

Futures contracts are important instruments for managing the price risk exposure of spot portfolios. To hedge the portfolio risk effectively, the hedging theory indicates that an optimal hedge ratio defined as an amount of futures position that is undertaken for each unit of the underlying spot should be adopted. Over years, a number of studies have developed econometric models for estimating the hedge ratio. While earlier studies have restricted the ratio to be constant over time (Ederington, 1979), recent studies recognize that spot-futures distribution is time-varying, hence the hedge ratio should be time-dependent (Baillie & Myers, 1991; Moschini & Myers, 2002). To address this issue, multivariate generalized autoregressive conditional heteroscedasticity (GARCH) models are usually employed in estimating the conditional second moments that are relevant for hedge ratio estimation (Myers, 1991; Kroner & Sultan, 1993; Park & Switzer, 1995; Brooks et al., 2002; Lien & Yang, 2006). Since the hedge ratio changes as new information arrives to the markets, generally, this realistic ratio tends to outperform the static ordinary least squares (OLS) one in terms of risk reduction size.

To enhance the hedging performance empirically, recent studies have considered two classes of augmented GARCH for overcoming some of the limitations in the standard models. One is to allow for potential regime shifts between spot and futures dynamics under different market scenarios, because it is observed that standard GARCH tends to impute high levels of volatility persistence due to structure breaks in the volatility process (Lamoureux & Lastrapes, 1990). Henceforth Markov regime switching (MRS) models are developed for effective hedging (Alizadeh & Nomikos, 2004; Lee & Yoder, 2007a, 2007b; Alizadeh et al., 2008; Lee, 2009, 2010). The empirical results show that one may obtain more reliable hedge ratio estimates when the joint distribution can be switched stochastically between regimes, as a result, better hedging performance is obtained.

In addition to augmenting standard GARCH models using MRS techniques, recent studies also indicate that modeling and forecasting of a GARCH can be improved by incorporating finer intraday prices (Engle, 2002; Andersen et al., 2003; Koopman et al., 2005). This is because the realized variance (RV) by summing intraday squared returns provides more information about current level of integrated variance (IV) relative to that of using daily squared returns (Andersen et al., 2001; Barndorff-Nielsen & Shephard, 2002). Lai and Sheu (2010) demonstrated that, in the S&P 500 equity index market, both the statistical and economic hedging effectiveness are substantially improved with the use of intraday returns, relative to the use of daily returns.

This study contributes to this line of research by developing, for the first time, a MRS-GARCH model with finer RV (henceforth MRS-GARCH-X) for hedge ratio estimation. This MRS-GARCH-X hedging model accommodates the techniques of time-variation, state-dependency, and precise RV in estimating the hedge ratio, which itself in spirit generalizes the GARCH model of Baillie and Myers (1991), the MRS-GARCH model of Lee (2007a), and the GARCH-X model of Lai and Sheu (2010) for hedging. To ensure the benefits of a hedge using this MRS-GARCH-X model, the highly-traded NASDAQ 100 equity index futures traded on Chicago Mercantile Exchange (CME) is applied for the investigations. Empirical evidences suggest that the in-sample fitting of spot-futures distribution improves with the use of this proposed model, relative to the use of reduced GARCH, MRS-GARCH, and GARCH-X models. In addition to in-sample fitting, out-of-sample investigations are also carried out because the hedging decision has to be made ex-ante. Consequently, a rolling window method is involved for the out-of-sample period to provide robust evidence on the usefulness of MRS-GARCH-X hedge ratios for long NASQAD 100 positions. The results indicate that the MRS-GARCH-X accommodating the techniques of time-variation, state-dependency, and precise RV provides superior performance in terms of both risk reduction and utility growth sizes. The benefit of using the MRS-GARCH-X model for effective hedging is clearly supported.

The remainder of this study proceeds as follows. In the next section, the MRS-GARCH-X model is introduced for hedging, and is related to the reduced GARCH, MRS-GARCH, and GARCH-X models. Section 3 describes the data; and Section 4 provides empirical results. Finally, Section 5 concludes this study.

MRS-GARCH-X MODEL AND HEDGING

Multivariate GARCH models are widely adopted in estimating the dynamic hedge ratio.[1] This study considers using a BEKK specification for GARCH specification, because it allows for rich and flexible dynamics for the conditional second moments. An augmented GARCH model with MRS and RV components is provided to supplement the standard GARCH model for describing spot-futures distribution more realistically. Then the hedge ratio estimates are directly obtained from the forecasted conditional covariance matrix by the MRS-GARCH-X model.

It is documented that standard GARCH models fail to statistically fit the observed price data (Carnero et al. 2004). Hence, a plenty of augmented GARCH models has been proposed, such as MRS-GARCH (Lee & Yoder, 2007a, 2007b) or GARCH-X

[1] For example, Baillie and Myers (1991) adopted the vector error correction (VEC) framework of Bollerslev et al. (1988) for hedging commodities; Kroner and Sultan (1993) adopted the constant conditional correlation (CCC) framework of Bollerslev (1990) for hedging foreign currency; Brooks et al. (2002) adopted the BEKK framework of Engle and Kroner (1995) for hedging equity index; Lien and Yang (2006) adopted the dynamic conditional correlation (DCC) framework of Tse and Tsui (2002) for hedging currency; Lai and Sheu (2011) adopted the asymmetric DCC framework of Cappiello et al. (2006) for hedging equity index portfolios.

(Engle, 2002; Visser, 2011) models. Assume that the state-dependent spot and futures returns in a generalized MRS-GARCH-X model can be specified as

$$\begin{array}{l} r_{s,t} = \alpha_{s,st} + u_{s,t,st} \\ r_{f,t} = \alpha_{f,st} + u_{f,t,st} \end{array}, \quad \mathbf{u}_{t,st} = \begin{bmatrix} u_{s,t,st} \\ u_{f,t,st} \end{bmatrix} | \, \Omega_{t-1} \sim N(\mathbf{0}, \mathbf{H}_{t,st}) \tag{1}$$

where $\alpha_{s,st}$ and $\alpha_{f,st}$ are state-dependent conditional means of spot and futures returns, respectively, $\mathbf{u}_{t,st}$ is a state-dependent vector of Gaussian white noise process with time-varying 2×2 positive definite covariance matrix $\mathbf{H}_{t,st}$, and Ω_{t-1} is the information set available on day $t-1$. It means that the conditional means, noises, and covariance matrix in MRS-GARCH-X depend on the market regime at time t represented by the unobserved state variable $st = \{1,2\}$, which is assumed to follow a two-state, first order Markov process with the following transition probabilities:

$$\begin{array}{l} \Pr(st = 1 \mid st = 1) = P \\ \Pr(st = 2 \mid st = 2) = Q \end{array} \tag{2}$$

where P and Q are assumed to remain constant between successive periods.

Moreover, the state-dependent variance/covariance matrix is assumed to follow a GARCH formulation with all orders set to 1, which is given by

$$\begin{aligned} \mathbf{H}_{t.st} &= \begin{bmatrix} h_{s,t,st} & h_{sf,t,st} \\ h_{sf,t,st} & h_{f,t,st} \end{bmatrix} = \mathbf{C}'_{st}\mathbf{C}_{st} + \mathbf{G}'_{st}\mathbf{H}_{t-1}\mathbf{G}_{st} + \mathbf{A}'_{st}\mathbf{X}_{t-1}\mathbf{A}_{st} \\ &= \begin{bmatrix} c_{11,st} & c_{12,st} \\ 0 & c_{22,st} \end{bmatrix}' \begin{bmatrix} c_{11,st} & c_{12,st} \\ 0 & c_{22,st} \end{bmatrix} + \begin{bmatrix} g_{11,st} & g_{12,st} \\ g_{21,st} & g_{22,st} \end{bmatrix}' \mathbf{H}_{t-1} \begin{bmatrix} g_{11,st} & g_{12,st} \\ g_{21,st} & g_{22,st} \end{bmatrix} \\ &+ \begin{bmatrix} a_{11,st} & a_{12,st} \\ a_{21,st} & a_{22,st} \end{bmatrix}' \mathbf{X}_{t-1} \begin{bmatrix} a_{11,st} & a_{12,st} \\ a_{21,st} & a_{22,st} \end{bmatrix} \end{aligned} \tag{3}$$

for $st = \{1,2\}$, where $\mathbf{C}_{st}$, $\mathbf{G}_{st}$, and $\mathbf{A}_{st}$ are state-dependent parameter matrices. In this formulation, the state-dependent conditional (co-)variances are a function of past (co-)volatility proxies $\mathbf{X}_{t-1}$ and conditional (co-)variances $\mathbf{H}_{t-1}$. When $\mathbf{X}_{t-1}$ is specified as $\mathbf{u}_{t-1}\mathbf{u}'_{t-1}$, this defines a standard state-depend BEKK(1,1) model with daily returns (henceforth MRS-GARCH; see Lee & Yoder, 2007a; Alizadeh, et al, 2008), which is an augmented fully parameterized BEKK model (henceforth GARCH) of Engle and Kroner (1995).

Many financial data sets include intraday data in addition to the daily returns. Such data sets contain more information about the current level of volatility, so in principle it should be possible to improve standard GARCH models based on daily returns. This point is illustrated by Engle (2002), Koopman et al. (2005), and Visser (2011). They show that includes finer realized volatility measures in the GARCH equation (known as a GARCH-X model) is very useful for modeling and forecasting future volatility, because any of these realized measures is far more informative about the current level of volatility than is the squared return (Andersen et al, 2001; Barndorff-Nielsen & Shephard, 2002).

Realized variance (RV) is a commonly applied proxy for daily IV measurement. Usually, RV is defined as the sum of the intraday squared returns, $RV_t = \sum_m r_{t,m}^2$, where $r_{t,m}$ denotes the return over the m$^{\text{th}}$ intraday interval of day t. In the absence of microstructure noise, Barndorff-Nielsen and Shephard (2002), Andersen et al. (2003) indicated that RV is a consistent estimate of IV when the sampling observation diverges. Extending the results for a univariate process to a multivariate framework, Barndorff-Nielsen and Shephard (2004) define the realized covariance (RC) by summing up intraday cross-product returns, $RC_t = \sum_m r_{t,m}^i r_{t,m}^j$, where $r_{t,m}^i$ and $r_{t,m}^j$ respectively denote the return over the m$^{\text{th}}$ intraday interval for asset i and j of day t. It is shown that RC is consistent for daily integrated covariance (IC) measurement in a frictionless market. With the finer realized volatility proxies, a MRS-GARCH-X model for spot and futures is given by defining

$$\mathbf{X}_{t-1} = \begin{bmatrix} RV_{t-1}^s & RC_{t-1} \\ RC_{t-1} & RV_{t-1}^f \end{bmatrix} \tag{4}$$

in equation (3). Obviously, equation (3) combined with equation (4) will reduce to a state-independent GARCH-X model when a single regime process is assumed.

To solve the well-known path-dependency problem in the regime switching literature, this study integrates the state dependent variances and covariance by transferring them into path-independent counterparts. Following Gray (1996), the conditional variances can be recombined using the equations

$$h_{i,t} = p_{t,1}(\alpha_{i,1}^2 + h_{i,t,1}) + (1-p_{t,1})(\alpha_{i,2}^2 + h_{i,t,2}) - [p_{t,1}\alpha_{i,1} + (1-p_{t,1})\alpha_{i,2}]^2 \qquad (5)$$

for $i = \{s, f\}$, where $p_{t,1}$ is the probability of being in regime 1 at time t, defined as

$$\begin{aligned} p_{t,1} &= \Pr(st = 1 \mid \Omega_{t-1}) \\ &= P\left[\frac{f_{t-1,1}p_{t-1,1}}{f_{t-1,1}p_{t-1,1} + f_{t-1,2}(1-p_{t-1,1})}\right] \\ &\quad + (1-Q)\left[\frac{f_{t-1,2}p_{t-1,1}}{f_{t-1,1}p_{t-1,1} + f_{t-1,2}(1-p_{t-1,1})}\right] \end{aligned} \qquad (6)$$

where

$$f_{t,st} = f(\mathbf{R}_t \mid \Omega_{t-1}) = (2\pi)^{-1} \mid \mathbf{H}_{t,st} \mid^{-1/2} \exp\left\{-\frac{1}{2}\mathbf{u}'_{t,st}\mathbf{H}_{t,st}^{-1}\mathbf{u}_{t,st}\right\} \qquad (7)$$

and $\mathbf{R}_t = [r_{s,t}\ r_{f,t}]'$ is a vector of spot and futures returns at time t. Similar to the variances, Lee and Yoder (2007a) showed that the state dependent covariance can be expressed as

$$\begin{aligned} h_{sf,t} = {} & p_{t,1}[\alpha_{s,1}\alpha_{f,1} + h_{sf,t,1}] + (1-p_{t,1})[\alpha_{s,2}\alpha_{f,2} + h_{sf,t,2}] \\ & - [p_{t,1}\alpha_{s,1} + (1-p_{t,1})\alpha_{s,2}][p_{t,1}\alpha_{f,1} + (1-p_{t,1})\alpha_{f,2}] \end{aligned} \qquad (8)$$

Using the collapsing procedure at each time step (equations (5)-(8)), the MRS-GARCH-X model becomes path-independent and tractable. Thus, the parameters of MRS-GARCH-X can be estimated by maximizing the log-likelihood (LL) function

$$LLF(r_{s,t}, r_{f,t}; \theta) = \sum_{t=1}^{T} \log[p_{t,1} f_{t,1} + (1 - p_{t,1}) f_{t,2}] \quad (9)$$

where $\theta = \{\alpha_{st}, c_{st}, g_{st}, a_{st}, P, Q\}$ represents the vector of parameters to be estimated. Note that we restrict c_1, c_3, g_1 and a_1 to be positive and apply the covariance stationary condition in Engle and Kroner (1995) to satisfy positive definite and covariance stationary in $\mathbf{H}_{t,st}$ for each state. After obtaining the parameters' estimates, a one-step-ahead hedge ratio forecast for time $t+1$ given all the available information up to t can be calculated by

$$\hat{\beta}^*_{t+1} = \hat{h}_{sf,t+1} / \hat{h}_{f,t+1} \quad (10)$$

where the conditional variance forecast $\hat{h}_{f,t+1}$ and conditional covariance forecast $\hat{h}_{sf,t+1}$ are calculated for the collapsing procedure as presented in equations (5) and (8), respectively.

Estimating hedge ratio using the MRS-GARCH-X model outlined above further incorporates finer intraday information in standard MRS-GARCH hedging, and relaxes the assumption of constant parameters in the GARCH-X process so that the hedge ratio depends on the state that the market is in. Once the state variable is restricted to be one state, the MRS-GARCH-X and the MRS-GARCH, respectively, reduce to the GARCH-X and the GARCH without allowing for switching stochastically under different market conditions. It is also noted that the MRS-GARCH model is a special case of the MRS-GARCH-X model when $\mathbf{X}_{t-1}$ is specified as $\mathbf{u}_{t-1}\mathbf{u}'_{t-1}$. In this situation, collapsing the residuals of spot and futures returns is necessary in estimating the MRS-GARCH, as follows:

$$u_{i,t} = r_{i,t} - [p_{t,1}\alpha_{i,1} + (1 - p_{t,1})\alpha_{i,2}] \quad (11)$$

While the literature has documented that either MRS-GARCH or GARCH-X models overcome some of the limitations that standard GARCH models exhibit, one expects a MRS-GARCH-X model accommodating all the techniques of time-variation, state-dependency, and precise realized measures of volatility should provide better description in fitting the data relative to the traditional models. Consequently, it is expected that the MRS-GARCH-X hedging would be superior to the GARCH, MRS-GARCH, GARCH-X, as well as static OLS[2] hedging in terms of hedging performance.

DATA DESCRIPTION

The performance of optimal futures hedging using the MRS-GARCH-X, the MRS-GARCH, the GARCH-X, the GARCH, and OLS models are applied to the NASDAQ 100 equity index futures traded on CME, covering the period of July 1, 2003 to June 30, 2010 (1763 trading days). Tick Data Inc. provides a record of the time and price of every trade/quote revision for the futures as well as their underlying equity index. To construct a continuous price series for futures, the prices of nearby contracts are used and rolled to the next month on any given day when the trading volume of the current contract is exceeded. The procedure of Barndorff-Nielsen et al. (2009) is also applied to the tick-by-tick data sets, because there may be multiple price observations with the same time stamp. In this situation, they suggested using the median price instead.

Having constructed the continuous time series for the futures contracts prices, price records for the spot occurring after 3:00 PM are dropped from the dataset. This is because the spot market closes fifteen minutes earlier than the (floor) section for futures. This means that we model and forecast the variation of open-to-close (8:30

[2] The OLS hedge ratio is defined as the ratio of the unconditional covariance between cash and futures returns over the variance of futures returns. Ederington (1979) shown that this static hedge ratio is derived by minimizing the unconditional variance of hedged portfolio returns. In practice, this OLS hedge ratio is obtained by regressing spot return on futures return with intercept parameter in a simple regression model; this is equivalent to the slope parameter estimate.

AM to 3:00 PM) continuously compounded returns on NASDAQ 100 positions, assuming that the hedger concerns with the price risk when both markets are open.

In addition to obtaining daily logarithmic returns, daily realized volatility estimates are also relevant for MRS-GARCH-X hedging. This study considers a 15-min sampling procedure for calculating the realized quantities. Since the realized quantities constructed using all the tick-by-tick observations will result in a biased and inconsistent estimate of the true integrated variance when the market microstructure noise is presence, in practice, it is suggested to select a moderate frequency for a variance/bias trade-off. As a result, we partition the time horizon from 8:30 AM to 3:00 PM (hence 390 minutes) into several 15-min girds by finding the closest transaction prices before or equal to each grid-point time for each day and asset. With the 15-min prices, daily RV and RC estimates[3] are computed, and they are directly related to the daily open-to-close returns.

Panel A of Table I presents the summary statistics of daily returns, where the returns are calculated as the logarithmic difference between the closing price and the opening price of a day (8:30 AM to 3:00 PM). It is observed that the futures price is more volatile than the spot price, as evidenced by higher standard deviation and extreme observations. The daily returns are very leptokurtic and left-skewed that departs from the normality assumption. Thereby a quasi maximum likelihood estimator is employed for models' estimations. Panel B of Table I summarizes the daily RV and RC of spot and futures. As can be seen, daily squared (cross-product) returns are much noisier estimates for IV (IC) compared with 15-min RV and RC, as evidenced by larger standard deviations and extreme observations. It is also observed that the distribution of realized quantity is right-skewed and leptokurtic. Andersen et al. (2001) indicated that it can be transformed to Gaussian normal by using a

[3] Note that noisy overnight returns are not included in RV and RC estimation, because this would diminish the performance difference between the volatility proxies.

logarithm function. Having obtaining the daily returns and realized quantities, we then investigate the empirical performance of MRS-GARCH-X hedging.

<Table I is inserted about here>

EMPIRICAL RESULTS

The empirical fitting of MRS-GARCH-X as well as alternative models are presented in Table II. We conduct the estimation of all models using data from July 1, 2003 to June 30, 2010. First, the likelihood function provides valuable information in fitting the joint distribution. It is observed that the likelihood function value increases when daily squared returns are replaced by precise RV estimates. For example, the likelihood function value of MRS-GARCH-X model is about 14683, which creates additional 90 values than the value of MRS-GARCH model. This implies that the in-sample fitting on the joint distribution improves when informative RV estimates are utilized; illustrating that intraday price captures more current information about volatility modeling than those of using daily price. Second, the covariance stationary eigenvalues in the last row indicate different persistence on the price dynamics. For example, the maximum eigenvalue for GARCH-X model approximates unity, which is much higher than the value of 0.8945 using standard GARCH. Similarly, the traditional MRS-GARCH exhibits less persistence compared to the MRS-GARCH-X model at each state. Importantly, a high volatility state is associated with low persistence in the variance and vice versa. Third, from the estimated transition probabilities, we can calculate the duration of being in each state. The transition probabilities of MRS-GARCH-X are estimated as $P = 92.84\%$ and $Q = 66.07\%$; these indicate that the average expected duration of being in low volatility regime is about 14 (=1/(1-0.9284)) days compared to 3 (=1/(1-0.6607)) days in high volatility regime 2. Thus, high volatility state is less stable and is characterized by shorter duration compared to low volatility state.

<Table II is inserted about here>

Next, we turn to investigate the empirical performance of using MRS-GARCH-X model for hedging. To do this, we divide the data into two: the in-sample data from July 1, 2003 to June 30, 2008 (1259 trading days), and the out-of-sample data from July 1, 2008, to June 30, 2010 (504 trading days). The in-sample state-dependent hedge ratio are calculated using equation (10), after integrating out the unobserved variable as described in equations (5), (8), and (11). Since hedgers are more concerned with how well they can hedge their positions in the future, we mainly focus on the out-of-sample performance. Note that the assessment is implemented by estimating the model recursively, using only data up to the specific date. Figure 1 compares the one-step-ahead hedge ratio forecasts for that period examined. The hedging performance of the alternative models is exhibited in Panel B of Table III. In addition to the out-of-sample results, the in-sample results are also summarized in Panel A of Table III for comparisons.

<Table III and Figure 1 are inserted about here>

Focusing on the variance of these hedged portfolio returns firstly, the results indicate that the MRS-GARCH-X model performs the best among the competing models. The improvement of MRS-GARCH-X model reaches 98.33% and 98.40% in terms of in-sample and out-of-sample variance reduction sizes, respectively, as compared to unhedged spot position. The poor performance of traditional GARCH model illustrates the benefit of employing both the techniques of state-dependency and precise RV in estimating the hedge ratio. Besides assessing the variance reduction size of the models, investors should be more interested in knowing the economic benefit of using MRS-GARCH-X model for hedging. To formally assess the performance of these hedges, the mean-variance utility function as in Kroner and Sultan (1993) is considered in the comparisons, as follows:

$$U(E(r_{t+1}^p), \mathrm{var}(r_{t+1}^p); \hat{\beta}_{t+1}^*, \gamma) = E(r_{t+1}^p; \hat{\beta}_{t+1}^*) - \gamma\, \mathrm{var}(r_{t+1}^p; \hat{\beta}_{t+1}^*) \tag{12}$$

where γ represents the level of risk aversion for an investor. The results show that the MRS-GARCH-X model delivers the highest average daily utility relative to the competing models. In addition, the out-of-sample utility gains over the GARCH, GARCH-X, and MRS-GARCH models are about 1091, 201, and 571 basis points per annum (252 days). The usefulness of simultaneously combining the techniques of time-variation, state-dependency, and precise RV estimates for effective hedging is clearly supported.

CONCLUSIONS

This study develops a new MRS-GARCH-X model, which accommodates the techniques of time-variation, state-dependency, and precise RV techniques for effectiveness hedging. The empirical usage of the model is examined with the use of NASDAQ 100 futures data. The in-sample result indicates the statistically fitting for the joint distribution can be improved over the models without using all of the techniques. Hence, the dynamics for spot and futures becomes more realistically when the flexible MRS as well as the informative RV techniques are allowed on the GARCH specification for describing the joint distribution. The in-sample results and the out-of-sample results with daily rolling over show that, the MRS-GARCH-X model exhibits good performance in terms of both criteria of risk reduction and utility growth, indicating the empirical usefulness of using MRS-GARCH-X model for effective hedging.

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Table I

Summary statistics of daily open-to-close returns and volatilities for the NASDAQ 100 index markets[a]

Panel A: Daily open-to-close returns[b]			
	Spot	Futures	
Mean	-1.43E-4	-2.63E-4	
Std. dev.	0.0127	0.0129	
Skewness	-0.2906	-0.2878	
Kurtosis	7.4849	7.9706	
Minimum	-0.0785	-0.0823	
Maximum	0.0645	0.0734	
Panel B: Daily open-to-close volatilities[c]			
	Spot variance	Futures variance	Covariance
(A) Volatility estimation using daily returns			
Mean	1.60E-4	1.66E-4	1.62E-4
Std. dev.	4.08E-4	4.40E-4	4.20E-4
Skewness	7.5583	8.1759	7.8629
Kurtosis	79.9504	90.8089	85.3223
Minimum	0	0	-3.58E-5
Maximum	6.17E-3	6.77E-3	6.46E-3
(B) Volatility estimation using 15-min returns			
Mean	1.41E-4	1.54E-4	1.42E-4
Std. dev.	2.79E-4	2.99E-4	2.82E-4
Skewness	8.0476	7.4850	7.7658
Kurtosis	88.8455	74.5566	81.4674
Minimum	3.08E-6	4.55E-6	2.80E-6
Maximum	4.27E-3	4.29E-3	4.25E-3

[a] The sample period is from July 1, 2003 to June 30, 2010 (1763 trading days).

[b] Returns are calculated as the differences in the logarithm of daily open-to-close (8:30 AM to 15:00 PM) prices.

[c] Realized variance and covariance, respectively, are calculated by summing daily or 15-min squared and cross-product returns from 8:30 AM to 15:00 PM of a day.

Table II

Estimates of alternative GARCH models for the NASDAQ 100 index markets[a]

Model	GARCH	GARCH-X[b]	MRS-GARCH		MRS-GARCH-X[b]	
			Low	High	Low	High
Mean equation						
α_s	0.0003	-0.0002	0.0003	-0.0028	0.0008	-0.0051
	(11.19)[c]	(-6.01)	(1.15)	(-1.73)	(3.01)	(-3.69)
α_f	0.0002	-0.0003	0.0002	-0.0033	0.0008	-0.0056
	(13.77)	(-9.53)	(0.86)	(-2.08)	(2.87)	(-3.95)
Variance equation						
c_1	0.0021	0.0017	0.0015	0.0201	0.0003	0.0059
	(56.02)	(57.10)	(5.99)	(13.28)	(1.02)	(4.89)
c_2	0.0028	0.0021	0.0021	0.0206	0.0005	0.0058
	(21.05)	(67.65)	(8.32)	(13.58)	(1.67)	(4.91)
c_3	0.0002	0.0002	0.0001	0.0001	0.0003	0.0005
	(5.84)	(3.46)	(0.54)	(0.40)	(3.74)	(1.45)
g_1	0.0635	0.7164	0.8155	0.5483	0.8053	0.6146
	(69.07)	(866.31)	(27.07)	(14.72)	(26.11)	(9.83)
g_2	-0.5988	0.0753	0.0613	-0.3296	0.1614	0.0765
	(-129.36)	(75.68)	(3.14)	(-6.18)	(7.11)	(1.35)
g_3	0.8589	0.1788	0.1406	-0.3881	0.1357	-0.0026
	(730.93)	(213.62)	(4.73)	(-7.94)	(4.67)	(-0.05)
g_4	1.4844	0.8011	0.8757	0.5040	0.7654	0.5622
	(2613.65)	(816.73)	(46.48)	(12.33)	(32.42)	(8.52)
a_1	0.2979	0.5198	0.1917	0.1664	0.1827	0.6868
	(26.22)	(200.43)	(5.04)	(1.99)	(6.26)	(15.23)
a_2	0.5503	0.2100	0.2488	0.0246	-0.0149	-0.0181
	(272.95)	(72.15)	(7.12)	(0.18)	(-0.49)	(-0.27)
a_3	-0.5809	-0.0738	0.0435	0.0803	0.1522	0.0709
	(-750.54)	(-34.04)	(1.19)	(0.89)	(5.01)	(1.23)
a_4	-0.8969	0.2719	0.0274	0.1292	0.3924	0.7288
	(-1470.02)	(120.29)	(0.73)	(1.41)	(13.40)	(18.19)
P, Q	-	-	0.9699	0.7504	0.9284	0.6607
			(167.88)	(20.68)	(106.00)	(8.40)
LLF[d]	14485.24	14610.62		14593.01		14683.47
Covariance stationary eigenvalues[e]	0.8945	0.9999	0.9610	0.7921	0.9993	0.9288
	0.6069	0.7059	0.5631	0.1677	0.6768	0.8420
	0.6069	0.5063	0.6843	0.0670	0.4535	0.7706
	0.6609	0.7173	0.7000	0.1680	0.6685	0.8476

[a] The sample period is from July 1, 2003 to June 30, 2010 (1763 trading days).
[b] Note that RV estimates used by the GARCH-X models are computed using a 15-min sampling scheme.
[c] Figures in parentheses are t-ratios.
[d] LLF stands for log-likelihood function.
[e] See Proposition 2.7 in Engle and Kroner (1995).

Table III

Hedging effectiveness of alternative GARCH models[a]

	Variance[b]	Variance improvement of MRS-GARCH-X[c]	Utility[d]	Utility gains of MRS-GARCH-X over other hedging models[e]
Panel A: In-sample hedging effectiveness				
Unhedged	160.3096	98.33%	-642.6671	633.1796
OLS	2.7132	1.62%	-9.7049	0.2174
GARCH	2.8057	4.87%	-10.0368	0.5493
GARCH -X	2.6868	0.66%	-9.6082	0.1207
MRS-GARCH	2.7131	1.62%	-9.6245	0.1370
MRS-GARCH-X	**2.6692**		**-9.4875**	
Panel B: Out-of-sample hedging effectiveness				
Unhedged	319.7786	98.40%	-1280.2728	1261.8668
OLS	5.3954	5.07%	-19.6021	1.1961
GARCH	6.0621	15.51%	-22.7358	4.3298
GARCH -X	5.3144	3.62%	-19.2030	0.7971
MRS-GARCH	5.5411	7.56%	-20.6714	2.2654
MRS-GARCH-X	**5.1221**		**-18.4060**	

[a] The in-sample period is from July 1, 2003 to June 30, 2008 (1259 trading days); and, the out-of-sample evaluation period is from July 1, 2008 to June 30, 2010 (504 trading days).

[b] Variance denotes the variance of the hedged portfolio multiplied by 10^6. Figures in bold denote the best performing model for each criterion.

[c] Improvement of MRS-GARCH-X over other hedging models measures the incremental variance reduction of the MRS-GARCH-X over other models.

[d] Utility is the average daily utility for an investor with a mean-variance utility function and a coefficient of risk aversion of 4, multiplied by 10^4.

[e] Utility gains of MRS-GARCH-X over other hedging models measures the difference of the expected daily utility of MRS-GARCH-X and the expected daily utilities of other GARCH models.

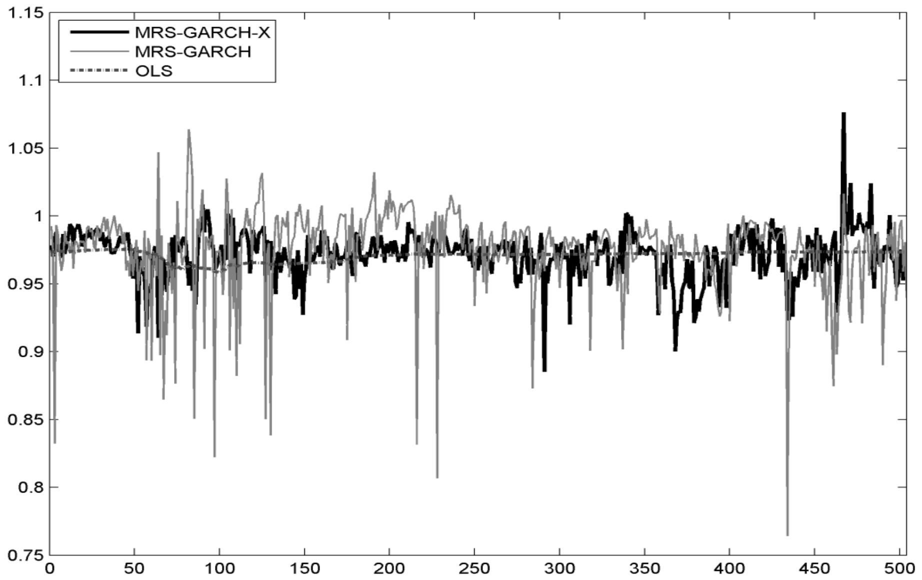

FIGURE 1. (a) Out-of-sample hedge ratios for the rolling MRS-GARCH and OLS models for the period of July 1, 2008 and June 30, 2010 (504 trading days).

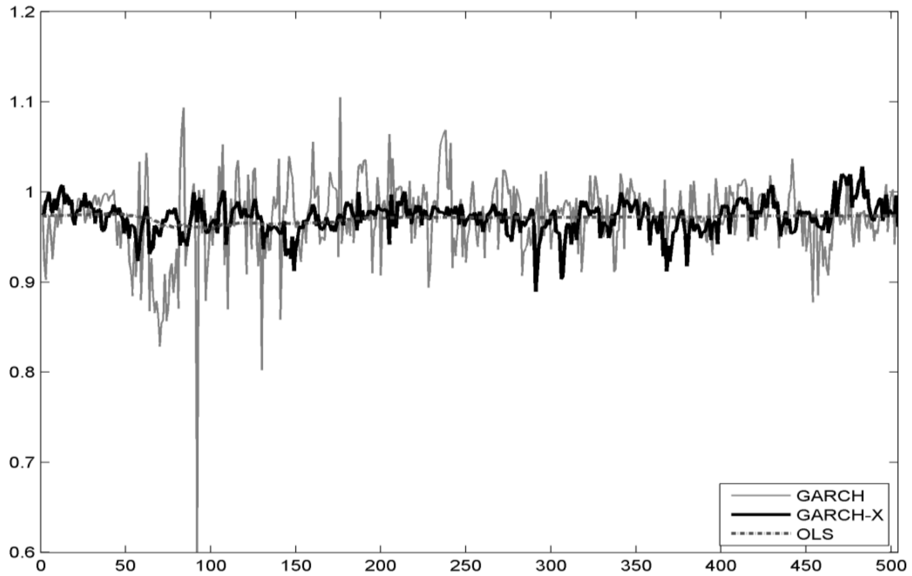

FIGURE 1. (b) Out-of-sample hedge ratios for the rolling GARCH and OLS models for the period of July 1, 2008 and June 30, 2010 (504 trading days).